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August 14, 2012

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This work was funded by the Center for the Commercialization of Innovative Transportation Technology at Northwestern University, a University Transportation Center Program of the Research and Innovative Technology Administration of USDOT through support from the Safe, Accountable, Flexible, Efficient Transportation Equity Act (SAFETEA-LU).

# Strategic Gang Scheduling for Railroad Maintenance

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**Abstract:** We address the railway track maintenance scheduling problem. The problem stems from the significant percentage of the annual budget invested by the railway industry for maintaining its railway tracks. The process requires consideration of human resource allocations (gangs), as well as effective logistics for equipment movement and routing around the rail network under time window constraints. We propose an efficient solution approach to minimize total costs incurred by the maintenance projects or jobs within a given planning horizon. This is accomplished by designing a job-time network model to capture feasible schedules under the constraints of job precedence and developing a mathematical programming heuristic to solve the underlying model. The key ingredient is an iterative process that extracts and then re-inserts jobs based on an integer programming model. Computational experiments show the capability of the proposed heuristic to schedule more than 1,000 jobs and more than 30 gangs.

**Keywords:** Railway Maintenance, Integer Programming, Heuristics

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## 1. Introduction

Since the inception of the railway industry during the Industrial Revolution, the industry has evolved into a vital means of transportation worldwide. The US railway industry nowadays owns an intensive infrastructure that requires constant maintenance and improvements. In the US, a railway owns the tracks and has to maintain them. Such a business model is the focus of this paper. In Europe, a track is shared by many railways, which require different maintenance agreements.

By taking into account its tens of thousands of miles of tracks, which are subject to wear and failures due to their extensive use, the railway industry must judiciously plan its day-to-day planning problems and the preceding tactical challenges. This fact is supported by the equipment and labor intensive processes typically conducted throughout a year for the maintenance of tracks, including both preventive and real-time maintenance.

According to TATA Consultancy Services (2002), the American Public Transportation Association (APTA) estimates that track maintenance expenditures comprise roughly 9% of total operating costs, from which nearly 80% corresponds to labor cost. In monetary terms these numbers may correspond to millions of dollars on annual maintenance projects (Judge, 2010).

Maintenance workers work in groups, called gangs, performing heavy labor duties such as installing track ties, driving track spikes, shoveling ballast and other similar maintenance track related activities. A gang consists of several members (between 10–100) with different responsibilities such as machine operators, termite welders, assistant extra gang foreman, or extra gang foreman. A gang beat or a beat section is a section of a track assigned to a gang to work on. During a year a gang moves from one beat section (a job) to another upon completion of the work at the section.

The railway maintenance scheduling problem (RMSP) is to find a sequence of jobs to be assigned to each gang over a time horizon (typically one year). Together with such an assignment, for each job a start and finish date must be determined. The goal is to minimize the overall cost while obeying various business requirements (e.g., certain jobs must be performed before other, a gang can only perform a job based on qualifications, etc.).

The RMSP is typically solved a few months before the start of the planning horizon, i.e., during the summer for the next calendar year.

A Class-1 US railway typically undertakes an intensive effort to determine and sustain annual rail maintenance schedules. This scheduling process is often labor intensive and leads to a costly and sub-optimal schedule. In this paper we propose a solution approach to efficiently solve the RMSP. We develop an algorithm that focuses on three specific issues: 1) minimize the total cost incurred by all track maintenance projects conducted throughout a given planning horizon, including cost of transition between jobs and cost of equipment movement between two locations, as well as cost of travel between a job location and the home-base of a gang, 2) provide practical feasible solutions that obey all business requirements, for example, it is required for gangs to work at a southern area during a winter season, 3) to be capable of computationally handling large-scale instances, such as those posed by Class-1 US railways.

More precisely, our solution methodology is based on a mathematical programming heuristic that is based on a job-time network model designed for each gang type. The network model basically captures feasible schedules, i.e., each schedule corresponds to a path (reverse not being true). As a result of solving the underlying model, our heuristic provides a maintenance schedule that outperforms current industry practice.

The RMSP differs from other maintenance problems by the heavy machinery being used and the large number of crew members that are involved in the track maintenance projects. One of the most distinct characteristics from its highway counterpart is the large spatial area to be covered by the whole maintenance schedule, i.e., it typically spans the entire railway network. In addition, ongoing track maintenance projects may imply curfew in certain sections and train schedules must be altered. Hence, a large number of specific business rules (side constraints) is usually imposed by railway companies to assure that the maintenance schedule is practically implementable, thus significantly increasing the complexity of this problem.

Existing approaches to gang scheduling either solve the entire problem directly by an optimization solver or iteratively over a subset of gangs. Due to the large number of jobs in our case, none of the existing approaches are applicable to the problem studied herein. We designed a novel heuristic that iteratively selects a subsets of gangs and jobs and then optimizes over the selection by means of a mathematical model. This model also introduces the flexibility of shifting in time the part of the solution that has not been selected.

The main contributions of this work are: 1) the design of a solution approach to solve the RMSP, which makes use of very large-scale neighborhood search ideas combined with mathematical programming, 2) a new integer programming based model for job re-insertion (the model allows for parts of schedules to be pushed back or forward in time), 3) the efficiency and scalability of the heuristic method, 4) and a large-scale case study consisting of more than 1,000 jobs, 10–100 gangs, a 365-day planning horizon, and all business requirements (the data comes from a Class-1 US railway).

The organisation of this paper is as follows. Section 2 provides the problem statement, in which business rules are described in detail. Section 3 introduces the job-time network model. Section 4 presents the proposed heuristic approach by describing its main modules, including a dynamic programming-based shortest path algorithm and the iterative mathematical programming-based steps (see Section 4.2). Section 5 reports the results of several computational analyses conducted on a case-study to show the capability of the proposed heuristic. Finally, concluding remarks are given in Section 6. The related work is discussed next.

## 1.1. Related Work

Despite the importance and complexity of railway gang scheduling problems, a survey of the literature reveals that not much relevant prior work has been reported in this area. Relevant works

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are due to Gorman and Kanet (2010), Gang (2006), and Peng et al. (2011). Gorman and Kanet (2010) proposed two modeling formulations to solve the RMSPP, namely a time-space network model and a job-scheduling network model. They provide a comparison between the two different formulations.

In contrast to the time-space network model, in which the number of gangs is variable, the job-scheduling formulation is based on the assumption that the number of gangs is known, thus resulting in a similar design to the network model presented herein. Due to the considerably large size of our instance, the models in Gorman and Kanet (2010) can not be applied to our case, i.e., a solver can not directly solve our instance and business requirements.

Gang (2006) proposed a preprocessing procedure to significantly reduce the size of the model of an unnamed railway for gang scheduling. The preprocessing technique basically reduces the size of the repositioning network and job time windows by combining (aggregating) jobs and eliminating variables and constraints. Consequently, a commercially available solver, CPLEX, is applied to a series of simpler problems, which are extensions of the original problem. We differ from Gang's work in that we tackle the complete model with a mathematical programming-based heuristic.

More recently, Peng et al. (2011) propose a heuristic solution approach to solve the RMSPP. The solution procedure is based on an iterative heuristic algorithm that is applied to a time-space network flow model. Contrary to our work, the heuristic method iteratively solves a sequence of subproblems with no more than two gangs each. Given the relatively small size of the resulting subproblems, they make use of a commercial software (CPLEX) to solve them, thus imposing severe limitations to the search space. In our case-study, even a model with two gangs can not be solved by a solver due to the sheer size of the problem.

Unlike in our proposed improvement stage, in which diverse analytical methods are applied (see Section 4.2), the solution improvement stage presented by Peng et al. (2011) is based on a random project swapping procedure that is called inside a loop. Due to the significant computational resources required by their heuristic method, coupled with the selection of two gangs to create the IP model, they prioritize the subproblems in such a way that a more promising improvement may be accomplished. The case-study reported in their work contains a total of 333 projects to be assigned to 21 gangs over a span of one year. This results in a significantly smaller test case than the one solved in our work.

To summarize, Gorman and Kanet (2010) and Gang (2006) solve models directly by optimization solvers, which is not applicable to our instance due to a large number of gangs and jobs. Peng et al. (2011) tackle the problem iteratively. Their methodology includes a step that does not scale to the problem of our size and thus their methodology is also not applicable.

Moreover, gang scheduling optimization takes as input a list of maintenance tasks. There are various publications on preventive maintenance (see Budai et al. (2004), Higgins (1998), Lake et al. (2000), and Percy and Alkali (2007)), railway track inspection (see Marino et al. (2007) and De Ruvo et al. (2005)), and track maintenance simulations (see, e.g., Simson et al. (2000) and the references therein). Furthermore, Higgins (1998) and Lake et al. (2000) also consider gang aspects, however, the planning horizon consists of only a few days.

On the surface there seems to be similarities between gang scheduling and maintenance equipment scheduling on rigs. Both involve heavy duty machines and equipment. Prior work on scheduling on rigs is due to Aloise et al. (2006), Korgaokar and Mitra (2006), and Trindade and Ochi (2005). In these publications, however, workforce is not taken into account. The focus is on providing a schedule for moving maintenance equipment from one platform to another. Gang scheduling is a more complex problem since it concurrently includes decisions on maintenance tasks, flow of heavy equipment, and workforce scheduling.

## 2. Problem Statement

The railway maintenance scheduling problem poses a complex optimization task that results in a combined job assignment-sequencing-timetabling problem. The aim is to find a strategic gang schedule to perform a wide range of maintenance projects within a given planning horizon. The resulting job-gang schedule is strongly governed by several regulatory and union rules, as well as by various physical factors. These aspects are discussed in detail in the subsequent sections.

A typical major Class-1 US railway deploys 4 types of gangs: rail, tie and surface (T&S), curve, and dual, which have to be scheduled over a span of one year. This results in a total of 10-100 gangs and several thousand jobs around the U.S. Each job has an underlying required gang type, which implies that a particular job can only be carried out by particular gangs. A certain subset of jobs is assigned to each gang with each job being specified by attributes (earliest start date, latest start date, duration, division, location, etc.). In addition, certain jobs must obey precedence constraints, e.g., work on ties must precede any work on the rails. From the operational perspective, it is not desirable to perform two jobs geographically close to each other during the same period since it could substantially disrupt train operations. Few occurrences of such situations create a more robust schedule.

Fig. 1 illustrates the problem as a resource assignment and job scheduling problem under timing coordination requirements. As gangs have to transit from one job to another in the same or different region, cost of transition between jobs and cost of moving equipment between two locations as well as cost of potential travel between a job location and home-base of gangs are considered to be minimized (gangs transition home during each weekend). In addition, equity with respect to the number of working days in the year is desirable.

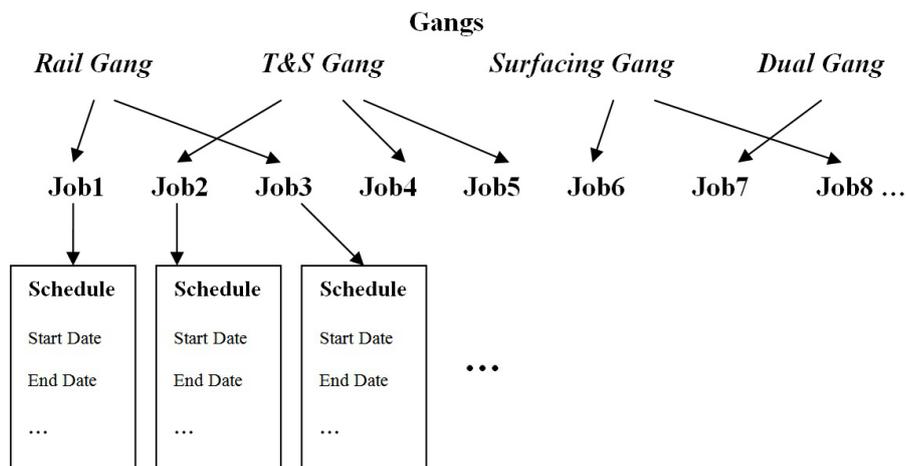


Figure 1 Relations between gangs and jobs.

### 2.1. Basic Business Rules and Cost Structure

Each gang has a unique type of work and a minimum and maximum number of working days in the year. Gangs work only a certain number of production days per week and at most a given number of productive hours per day. During winter, gangs should work in the south or coastal areas. Each gang has its own preferred geographical region, which is reflected in the cost.

The first job in the year of each gang is given, because at the end of the previous year, gangs may still have unfinished work in progress. Dual jobs precede T&S jobs, as well as do the curve jobs. However, while the latter is just desirable, the former is a hard rule.

Gangs get paid hourly. Gangs also enjoy a travel allowance when the distance exceeds a given number of miles. This distance is based on the travel distance from the current job to home and back to the next job. In addition to gang related costs, there is also a cost of moving equipment from one job to another. This cost is non-linear with respect to distance (beyond a certain distance, a locomotive must transport the equipment).

### 3. Network Model

A key observation when designing the network model for the RMSP is the existence of a unique set of exclusive jobs that can be assigned to each gang type (an attribute of a job specifies its type). Hence, we create individual networks for each gang type. Yet the presence of job precedence rules makes the problem inseparable among gang types (the fact that each job must be assigned to a gang within a gang type does not separate the problem by gangs). The networks are linked to each other by a supersource node and a supersink node, which have virtually unlimited supply and demand of gangs, respectively. Based on the business requirement, the supersource node corresponds to the first jobs of each gang since they are fixed. The supersource is also connected to the supersink to drain any excess supply.

Let  $G_v$  be the set of gang instances of type  $v$ . Then,  $\mathfrak{G}^g = (N^g, A^g)$  represents a network instance of type  $g \in G_v$ , where  $N^g$  and  $A^g$  are the node and arc sets, respectively. The network is composed of daily time as x-coordinate and job as y-coordinate. Each vertex or node  $(j, t)$  represents the available start time  $t$  (in daily increments) for each job  $j$  that can be processed by gang  $g$ . Let  $P^g$  be the set of paths (job node sequences) for gang  $g$  starting at job  $s^g$ , with an associated cost  $c_p^g$ .

Each node has a flag indicating an active or inactive status for a work week, being Friday through Sunday inactive nodes. Fig. 2 shows a job pool consisting of  $Job_1, Job_2, \dots, Job_n$ , where several (black and white) nodes were generated to represent the available start times for each job in the job pool. In the figure, white nodes are the inactive nodes corresponding to the weekend (including Friday as a day-off). Here, for example,  $Job_1$  has 3 active nodes to indicate that a gang can start the job at day 1, day 2, or day 3. Same argument for the remaining jobs. The arcs would then represent the transition in time from one job to another, until a path is completed. Fig. 3 shows the resulting scheme for a gang path, in which for each job  $u_i$ , its start and duration time,  $t_i$  and  $d_i$ , respectively, are defined.

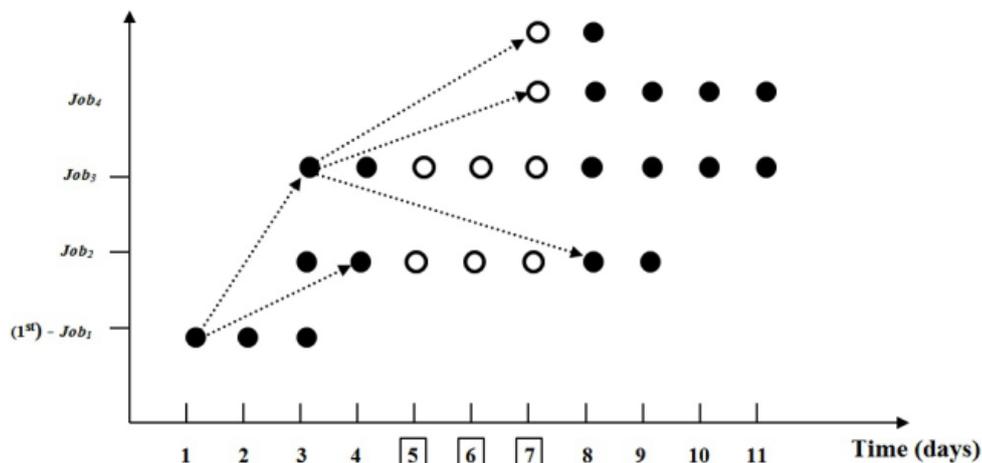
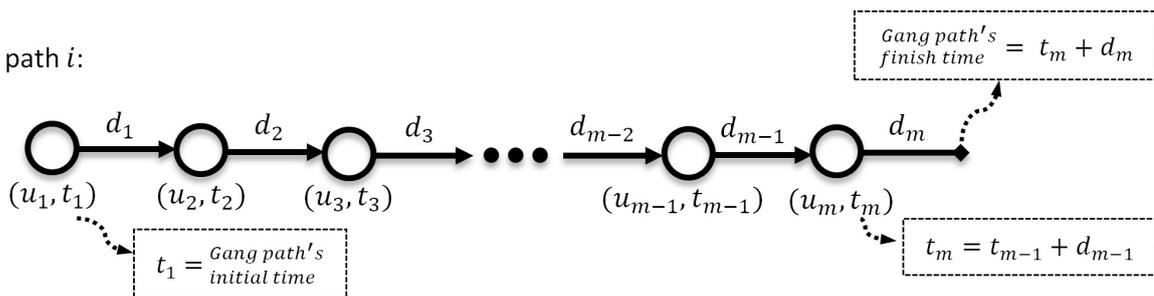


Figure 2 Construction of the network model for a gang.

Ex.:

Gang path  $i$ :



**Figure 3** Scheme of a job-gang path.

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**Algorithm 1** – *Solving\_RMSP\_by\_Alg1*

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**Input:** An instance of the RMSP.

Step 1: Construct the job-time network for each gang type.

Step 2: Find a path for each gang by running a DP-based shortest path algorithm.

Step 3: Phase 1: Find an initial solution by starting from solution obtained in Step 2.

Step 4: Phase 2: Apply an enhanced solution procedure to the initial point found in Phase 1.

**Output:** A favorable job-gang schedule for the RMSP.

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Concerning arc costs, they are calculated by travel miles and measured in dollar units. This is referred to the money paid to gangs traveling between the locations of two consecutive jobs. The formal definition of arc cost is given by: a) Travel costs between the location of job  $j$  and the home-town of the gang if the duration of job  $j$  contains weekends; b) travel costs between the location of job  $j$  and the home-town of the gang, plus the travel cost between the home-town of the gang and the location of the next job, if the time span of job  $j$ 's end time and the next job's start time contains weekends; and c) equipment movement costs from the location of job  $j$  to the location of the next job.

## 4. Solution Methodology

In this section, a two-phase heuristic approach is proposed to overcome the inherent complexity and relatively tight delivery date posed by RMSP-instances. Instead of employing a traditional local search strategy, we combine very large-scale neighborhood search ideas with mathematical programming. To this end, we develop a two-phase solution procedure. Basically, we construct a job-time network model for each gang type, and obtain a set of gang-paths by applying a shortest path algorithm using a dynamic programming method. As a result, the set of job-gang paths obtained is used in Phase 1 to heuristically assign jobs to gangs in order to find a feasible initial point. A Phase 2 is then applied to improve the initial solution by means of mathematical programming techniques. A summary of the solution framework is given in Algorithm 1.

The strategic idea behind Algorithm 1 stems from the fact that there is an exclusive job pool which can only be processed by gangs of a particular type. This decomposition significantly decreases the problem size and speeds up the solving time. Furthermore, other business rules such as job precedence and robustness are also relaxed. We propose two minimal constants,  $u_1$  and  $u_2$ , for the job precedence and robustness rules, respectively. These constants ensure that several strict rules are satisfied while other less important rules can be violated with no or insignificant penalty. Note that the robustness of the solution is nothing but the number of pairs of scheduled jobs located within a certain distance to each other and are concurrently performed. The full mathematical model is listed in Appendix A.

#### 4.1. Phase 1: Finding an initial solution

The first phase of the solution methodology is to find a feasible initial solution to the RMSP. This is accomplished by a heuristic searching procedure that basically comprises 4 modules, namely insertion, swap and increasing flexibility methods and a cycle overcome function, which are discussed next. The flowchart in Fig. 4 depicts *Phase 1* of our proposed solution framework.

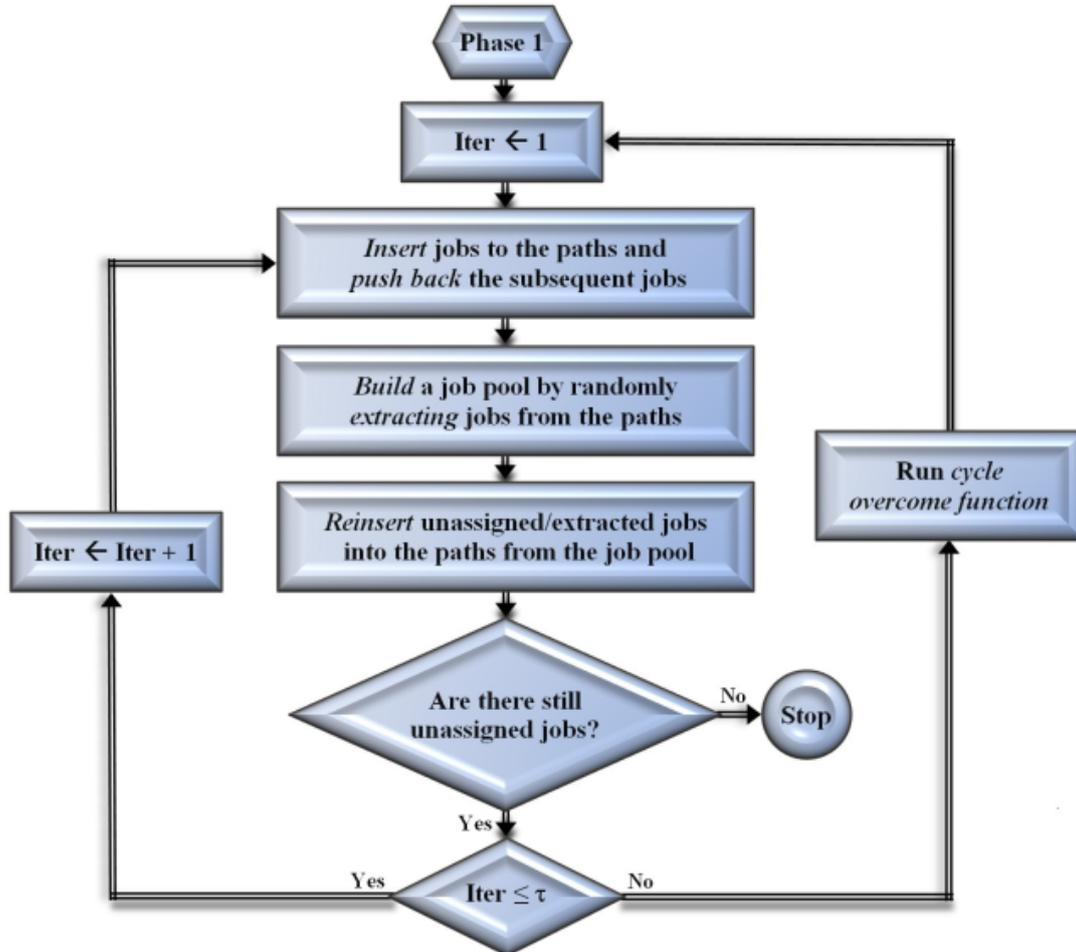


Figure 4 Solution framework, Phase 1: Finding a feasible initial point.

##### 4.1.1. A DP-based shortest path algorithm

Finding the optimal scheduling for each gang type is similar to find the shortest path with minimal cost in the job-time network. Hence, we apply a shortest path algorithm based on a dynamic programming method. Basically, the gang path to be found would cover all the jobs beginning from the first job, with a resulting cost being the cumulated cost of the last job node.

Once a gang finishes a job, the starting time for the next job is given by the finished job's end time plus the transition time. Consequently, the finished job's neighbors are those jobs whose available starting time are later than the calculated finished job's end time. As shown in Fig. 2, the first job ( $Job_1$ ) requires 2 working days to be done. Here, we call the completion day as duration. After finishing  $Job_1$ , the gang can move to either  $Job_2$  or  $Job_3$  immediately, or it can wait 4 working

days and then move to  $Job_4$ . Normally, the latter situation is impractical in real-life applications. Hence, only  $Job_1$ 's neighbors ( $Job_2$  and  $Job_3$ ) are considered.

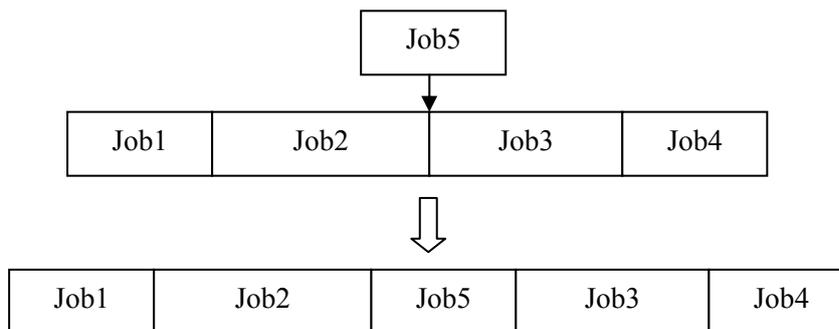
Balanced workload for the same gang type is an important requirement imposed by union regulations. As a strategy to handle this requirement, we do not choose the exact shortest path with the maximum number of jobs and smallest cumulated cost at the last job node. Instead we try to choose a path whose total duration  $sumDur$  (jobs completion time) falls in the range of  $[minWorkingDays, maxWorkingDays]$ , or,  $[minW, maxW]$  for simplicity. By doing so, we avoid violating several union rules since the shortest path may lead to one gang having a heavy workload, i.e., 250 days per year, while another gang having only 100 working days.

If no such path exists, a penalty cost occurs by incorporating the penalty multiplier  $(1 + k * balancePenalty)$ , with  $k$  as a constant, and  $balancePenalty = \frac{sumDur - maxW}{maxW - minW}$  if  $sumDur > maxW$  or  $balancePenalty = \frac{minW - sumDur}{maxW - minW}$  if  $sumDur < minW$ , i.e., the duration of the path exceeds the maximal working time or is less than the minimal working time, respectively. The optimality condition for choosing a path is to follow the rule that the optimal path should have at least  $2/3$  times the number of jobs that the longest path in the current network has.

#### 4.1.2. Insertion procedure

As mentioned above, several kinds of soft and hard constraints make the problem harder to solve. For example, gangs have a limited amount of working days per year, whereas the available jobs have a restricted time window. In addition, a set of precedence and robustness rules is imposed to certain jobs. Hence, the paths provided by the shortest path algorithm may not be suitable to schedule all possible jobs, thus leading to a pool of unassigned jobs.

To overcome this, the remaining unassigned jobs are inserted in the current paths (i.e., optimal paths for the corresponding gang types). Here, subsequent assigned jobs are pushed back after insertion of unassigned jobs. Fig. 5 shows the insertion procedure of unassigned  $Job_5$  in front of  $Job_3$ . The optimality condition for the position of insertion is to choose the smallest increment cost of the path among all possible positions and all possible paths for gangs of the same type. As an imperative requirement to successfully perform this insertion, the modified path must meet all imposed requirements, including job time windows, precedence rules and robustness.



**Figure 5** Insertion of unassigned jobs. Unassigned job  $Job_5$  is inserted before job  $Job_3$ .

Due to the very tight time windows of the jobs, the insertion procedure is somehow limited. Pushing back or forward those jobs already assigned to paths may cause violation of such time constraints, and thus, several unassigned jobs may remain unassigned after this stage. To overcome this, in the next section we introduce a swap and extraction method that increases the flexibility of the current paths to enforce the assignment of all remaining unassigned jobs.

### 4.1.3. Swap method

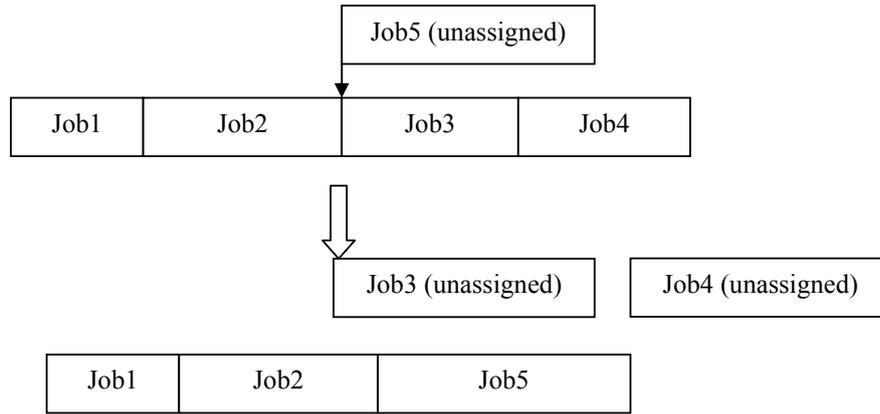
Since some unassigned jobs may remain in the pool of unassigned jobs after the insertion procedure, we apply a two-stage procedure. First, we strategically create enough room or ‘holes’ in the current paths by extracting those assigned jobs that can be swapped for unassigned jobs. Second, we insert the unassigned jobs and label the recently extracted job as unassigned. The key idea is to interfere with the smallest number of assigned jobs.

Fig. 6 shows the undone  $Job_5$  being swapped out for  $Job_3$  and  $Job_4$ , which were previously assigned to the path. As a result,  $Job_3$  and  $Job_4$  become undone jobs.

We design a procedure to determine the positions of swapping jobs, and thus increasing the flexibility of the paths, based on the  $curRatio$  function given by:

$$curRatio = \frac{incrementCost \times timeWindowLength2 \times curNumJobs}{timeWindowLength},$$

where  $timeWindowLength = \sum_{\text{in swapped jobs}} latestStartTime - earliestStartTime$  is the sum of time window length for all jobs that are to be swapped out in the path, and  $timeWindowLength2 = \sum_{\text{in swapped jobs}} latestStartTime - end$  is the sum of the difference between latest start time and end time for all jobs that are to be swapped out in the path. The larger the  $timeWindowLength$ , the more flexible the jobs are. The smaller the  $timeWindowLength2$ , the less flexible the jobs are.



**Figure 6** Swap of unassigned and assigned jobs. Unassigned job  $Job_5$  replaces jobs  $Job_3$  and  $Job_4$ .

### 4.1.4. Overcoming the cyclic problem

Running insertion and swap procedures for unassigned jobs lead to the successful insertion of more jobs. However, these procedures may cause the occurrence of cycle problems due to several unassigned jobs originally inserted in the paths may be swapped out again as the algorithm progresses. This is a result of the inflexibility of the paths observed at the early iterations, i.e., the path has many jobs that are scheduled close to their  $latestStartTime$ , thus the insertion and swap can happen only in a few positions. In order to make paths more flexible, a sub-routine called *MeetDueDateJobOut* is designed to concurrently extract a set of jobs at random, under the condition that the path remains valid and no constraints are violated.

Note that each iteration includes one insertion and one swap. The former largely decreases the number of unassigned jobs in the job pool, and the latter slightly increases the number of unassigned jobs in the path. Since the *MeetDueDateJobOut* sub-routine significantly increases the number of unassigned jobs, it is run every  $\tau$  iterations (typically  $\tau=10$ ) until Phase 1 converges.

**Algorithm 2** – *ImprovingStage\_of\_Alg1***Input:** The initial point provided by Phase 1 of Algorithm 1.

Set of job-time networks, predetermined gang paths and data set of unassigned jobs.

Step 0: Iter  $\leftarrow$  0**repeat**Step 1: Randomly *extract* assigned jobs from the paths.Step 2: *Recombine* extracted/uncovered nodes into a pool of subsequences.Step 3: *Form* and *solve* an ILP model to optimally create subsequences of nodes.Step 4: *Incorporate* the optimal solution provided by the reallocation ILP into the current solution.Step 5: Iter  $\leftarrow$  Iter + 1**until** Iter  $\geq$   $\alpha$ **Output:** An improved gang scheduling for the RMSP.**4.2. Phase 2: An enhanced solution procedure**

In this section we propose a more sophisticated procedure that significantly improves the initial solution found in Phase 1 by covering unassigned jobs.

Since each gang has limited working days for the year and each available job has a restricted time window, Phase 1 cannot schedule all the possible jobs. Hence, Phase 2 of the Algorithm 1 includes an interchange procedure that iteratively improves the solution by scheduling all the jobs and assuring to find a no worse solution than the initial solution.

The basic idea of Phase 2 is to randomly select certain nodes to be extracted from the starting solution (see Fig. 7), and then derive sequences of nodes, including extracted nodes and uncovered nodes, to reinsert them into the short-cut solution. Next, we formulate an integer program, and solve it by applying an ILP solver. The IP-solution is then incorporated back into the original solution. This procedure is repeated until an iteration limit is reached or the solution is significantly improved. A summary of Phase 2 of the solution framework is given in Algorithm 2.

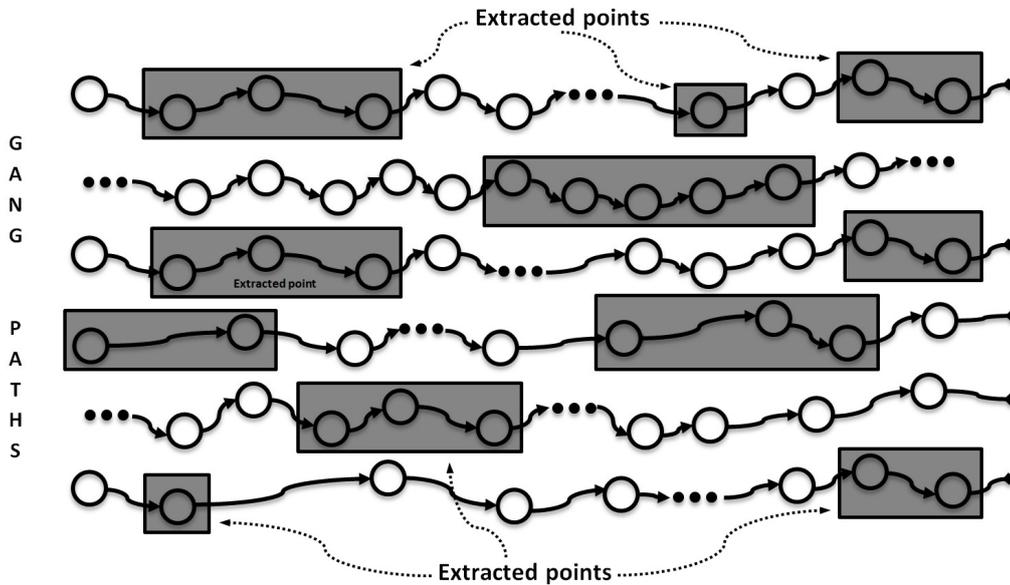


Figure 7 Points selected for extraction.

### 4.2.1. Extraction

In this step, as shown in Fig. 7, a number of job sequences are extracted randomly from the gang paths. Basically, we first determine a specific number of job sequences to be extracted from each gang path. The sequences are composed of a random number of job nodes, and are extracted from specific points that are strategically spanned over the whole path. By doing so, we create enough available space (several holes in the path) for the new sequences of job nodes.

Overlapping sequences are permitted for extraction since they basically imply that two or more specific origin points overlapped when the random number of job nodes of each sequence has been chosen. In this case, they are treated as a unique entity of nodes.

### 4.2.2. Recombination

In this step, we use all the extracted and uncovered nodes (see Fig. 8) to create a pool of subsequences that can be potentially reinserted into the solution. To make the problem simpler and improve the performance, we only consider short sequences of nodes. The business rules, such as job precedence and robustness, are also considered here.

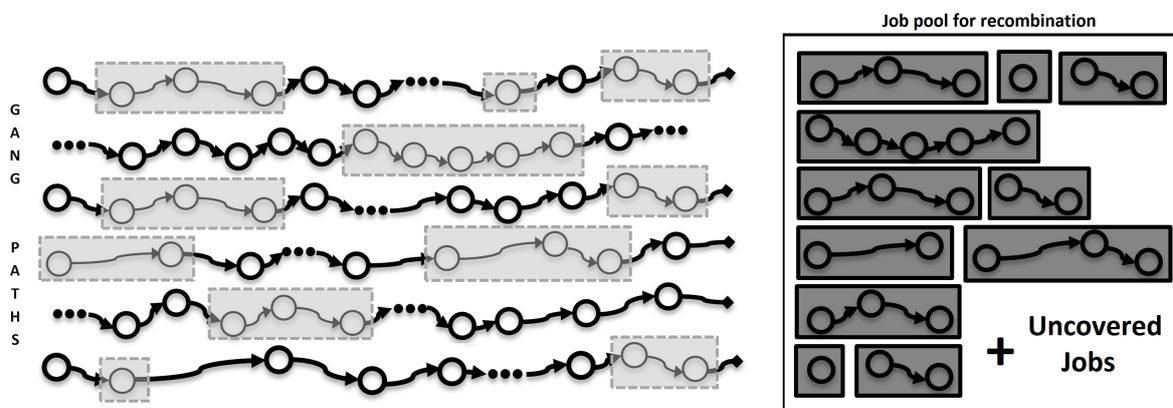


Figure 8 Pool of extracted/uncovered jobs for recombination.

### 4.2.3. Reallocation Model

In this step, we form an insertion ILP model. By solving the ILP model, we optimally reallocate the extracted and uncovered nodes from the previous step. Note that the model is always feasible since the nodes may always be reinserted back to the original locations. The detailed model is provided in Appendix A.

We introduce both binary decision variables to indicate whether certain sequences can be inserted at some insertion point  $i$ , and integer decision variables to indicate the shifting time of the job sequences at the short-cut solution after nodes extraction. Note that it is critical to introduce shifting time variables since at some insertion point, the starting time of the subsequent job sequence must be pushed back or forward to, respectively, enlarge or shrink the time window in order to allocate the new sequence of nodes.

Existing approaches to gang scheduling either solve the entire problem directly by an optimization solver or iteratively over a subset of gangs. Due to the large number of jobs to be scheduled in the real practice, none of the existing approaches are applicable to the problem studied herein. The ILP model is essentially an assignment problem with the additional constraints corresponding to job precedence rules, robustness and job time windows, that aims at minimizing the total

cost. Since the complete ILP model has a large number of columns, we thus solve it by randomly choosing a predetermined number of columns from the entire set of columns.

#### 4.2.4. Reinsertion

We incorporate the optimal results obtained from the reallocation ILP model into the solution using specially designed insertion operations. This reinsertion step marks the end of an iteration. The entire series of steps will be repeated until a number  $\alpha$  of iterations (typically  $\alpha=5$ ) is reached.

## 5. Numerical results

The computational results reported herein correspond to a large-scale real-world test case consisting of more than 1,000 jobs, 10–100 gangs with 4 gang types over a 365-day planning horizon. Data, while being supplied by a major Class-1 US railway, are treated with confidentiality and their disclosure is somehow limited. The solution procedures were developed in Visual Studio C++ v.2008 and run on a 2.4 GHz Intel(R) processor with 2 GByte RAM under MS-Windows operating system.

The purpose of the computational experiments is twofold. First, we evaluate the quality of the gang schedule provided by Algorithm 1 by comparing it to the original schedule performed by the current industry practice. Second, we assess the performance of our heuristic method.

Table 1 shows the comparison analysis between 3 different solutions provided by our heuristic and the baseline solution currently used by the railway. The only difference among the three solutions is the weight between the equipment movement cost and the remaining cost components. For each corresponding solution type shown in Column 1, the total cost, travel allowance, equipment movement (EM) cost and robustness are shown in the subsequent columns. The values correspond to the relative improvement of Alg1-solutions over baseline-solution, which are provided accordingly for each concept in Table 1. Soft violations are included in the last column.

**Table 1** RI of Alg1-solutions over the baseline-solution.

Solution	Total cost (%)	Travel allowance (%)	EM cost (%)	Robustness (%)	Soft Violations
A	18.8	22.9	-6.0	34.0	0
B	11.1	13.3	-2.4	58.3	0
C	5.3	8.1	-12.0	37.8	0

From the results shown in Table 1, we can make three key observations. First, all Alg1-solutions outperform the baseline-solution with a relative improvement up to 18.8%, which may result, in monetary terms, in millions of dollars of annual saving for the railway industry. Second, soft precedence rule violations were reduced by 100%. Third, Alg1-solution B achieved the best robustness, yet its total cost is outperformed by Alg1-solution A. Here, a trade off between total cost and robustness may become an important factor for managers to take into account when solving the RMSP.

According to the business requirements, the robustness is a soft rule and we therefore apply a penalty to those jobs that violate it. In general, the larger the robustness penalty, the smaller the violation will be. However, since the insertion, swap and increase flexibility procedures do not take into account any robustness rule, the penalty and number of violations are not exactly negatively correlated. Hence, a sensitivity analysis is conducted to evaluate the effect of this penalty on total cost estimates. Table 2 shows the outcome of conducting such analysis.

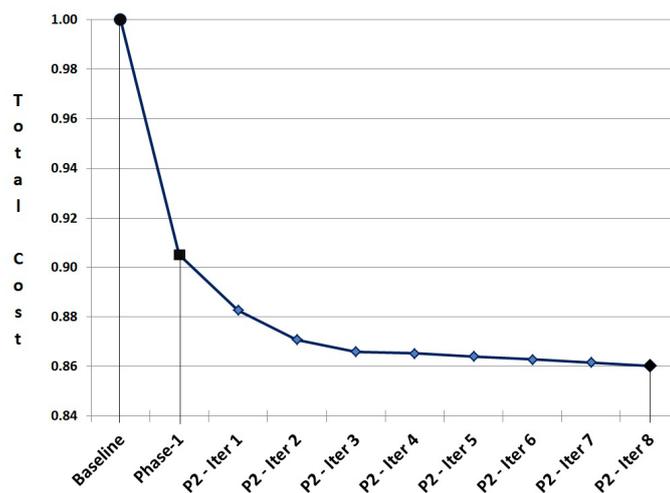
**Table 2** Sensitivity analysis on the robustness of the solution.

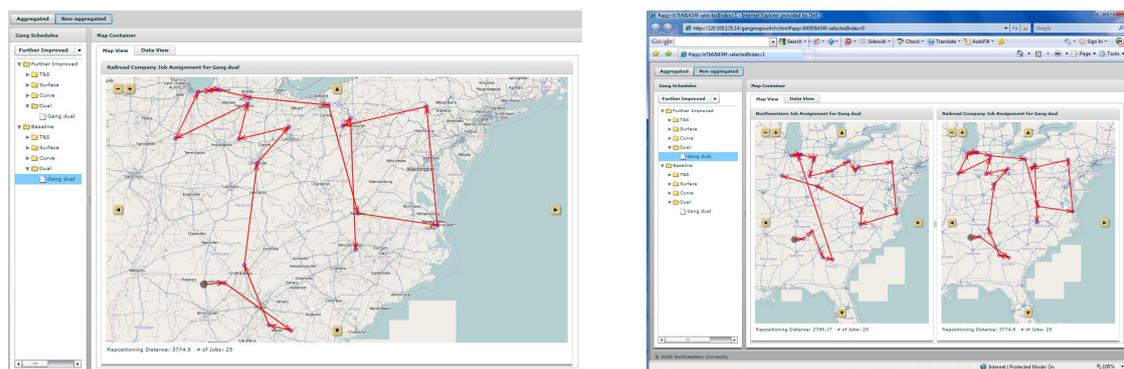
Robustness penalty	Robustness violation	Soft violation	Total cost (RI%)
Baseline	1.00	$\geq 40$	–
500	0.68	0	11.3%
1000	0.65	0	10.1%
<b>1500</b>	<b>0.69</b>	<b>0</b>	<b>14.3%</b>
2000	0.72	0	6.3%
2500	0.69	0	8.6%
3000	0.67	0	4.9%

From Table 2, we observe that the best solution achieved by our heuristic is provided by a robustness penalty set to 1500. Here, both total costs and robustness violations were improved by more than 14% and 30%, respectively, when compared to the baseline solution. More precisely, in relative terms, robustness rule violations dropped from 1.0 to 0.69, soft precedence rule violations were reduced by 100%, and total costs were reduced 14.3%.

Several conclusions were drawn based on a more detailed analysis when comparing actual gang schedules provided by our heuristic and the baseline. First, the total duration for gangs of the same type was similar for both schedules. Second, the divergence of balanced workload for each gang, a very significant issue for the union rules, turned out to be more favorable with our proposed schedule.

Fig. 9 shows the performance of Phase-1 and Phase-2 (P2) of Algorithm 1. From the figure, we can observe that a significant decrease in the total cost is obtained by Phase-1 (initial solution) when compared to the baseline solution. Similarly, our initial solution is significantly improved by the first iteration of Algorithm 1’s Phase-2. We observe that total costs keep decreasing until no significant improvement is achieved after 8 iterations. More precisely, the total cost decreases relatively slowly after iteration 3. Concerning the running time, Phase-1 required roughly 1.0 CPU-time hour to provide an initial feasible solution, whereas Phase-2 finished after 8 iterations in about 1.5 CPU-time hours. This proves the capability of our heuristic approach when solving RMSP’s large-scale test cases.

**Figure 9** Performance of Phase 1 and Phase 2 of Algorithm 1.



**Figure 10** Web-based interface: Schedule of a gang and graphical comparison of two schedules.

The front-end of the system, as shown in Fig. 10, is a web-based interface allowing the user to visually compare various solutions by displaying on a geographical map all maintenance jobs together with the flow of gangs.

## 6. Conclusions

We have proposed a heuristic solution approach for tackling the railway track maintenance scheduling problem. Our solution methodology consisted of 5 main modules, namely a network node constructor, a DP-based shortest path procedure, and the insertion, swap and increase flexibility methods. Basically, our algorithm applies inside a loop a heuristic method designed to find balanced workload paths for each gang. This is accomplished by choosing a suitable path after running a DP-based shortest path algorithm.

As a more general conclusion, the heuristic methods developed in this work are totally new and very suitable for optimizing scheduling problems with time window constraints. We designed a robust code that overcomes cyclic problems by using different penalty functions and making the paths more flexible such that all jobs can be assigned and scheduled in the paths.

Several comparative analyses were conducted to support the capability of the proposed heuristic. The gang scheduling provided by our heuristic outperforms above 14% the baseline schedule performed by the current industry practice. This is a very promising result since it significantly reduced the overall maintenance cost for the railway industry. Finally, our heuristic proved to be effective and efficient while requiring about 2.5 CPU-time hours to schedule more than 1,000 maintenance projects with time windows under the constraints of job precedence and robustness rules. This is reasonable for the railway managers to find favorable gang maintenance schedules.

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## Appendix

### A. Reinsertion ILP

We formulate the insertion gang scheduling problem as a network formulation. Given a set of gangs, and a set of jobs to insert, whereby each gang can perform a job in a given amount of time and at a given cost, the objective is to “attach” each job to an existing path (gang schedule), and timetable them to minimize total cost. For simplicity, we assume that the duration of jobs is independent of gangs.

#### A.1. Nomenclature:

The following notation is defined for each gang type  $v$ . As defined in Section 3, a job node is represented by  $(u, t)$ , where  $u$  is the job, and  $t$  is the starting time. A path  $p$  consists of a sequence of job nodes, i.e.,  $p \equiv \{(u_1, t_1), (u_2, t_2), \dots, (u_n, t_n)\}$ , with  $t_1 \leq t_2 \leq \dots \leq t_n$  and all jobs being different. For path  $p$  and time  $\bar{t}$ , we denote by  $p + \bar{t}$  the path defined by  $\{(u_1, \bar{t}), (u_2, t_2 + \bar{t} - t_1), \dots, (u_n, t_n + \bar{t} - t_1)\}$ . This entails to managing path  $p$  to time  $\bar{t}$  while preserving the job sequence.

#### Sets and parameters:

$\mathcal{N}$ : set of all job nodes;

$\mathcal{F}$ : set of extracted and uncovered job nodes;

$\mathcal{O}$ : set of job nodes remaining in the network after extraction,  $\mathcal{O} = \mathcal{N} - \mathcal{F}$ ;

$\mathcal{P}$ : set of paths constructed based on extracted/uncovered jobs;

$\mathcal{I}$ : set of insertion points;

$fj(p)$ : first job of path  $p$ ;

$lj(p)$ : last job of path  $p$ ;

$\mathcal{J}^i$ : set of starting nodes available for insertion point  $i$ ;

$T$ : total number of working periods (in days);

$\gamma_s^i$ : cost due to insertion of path  $s \in \mathcal{P}$  at insertion point  $i$ ;

$Sim$ : set of job pairs  $(u, w)$  such that it is not desirable for jobs  $u$  and  $w$  to be processed simultaneously;  $u, w \in \mathcal{N}$ ;

$Pre$ : set of job pairs  $(u, w)$  such that job  $u$  must be processed before job  $w$ ;  $u, w \in \mathcal{N}$ ;

$D_u$ : duration (days) of job  $u$ ;

$sd(p)$ : start time of path  $p$ ;

$ed(p)$ : end time of path  $p$ ;

$jobs(p)$ : set of jobs associated with path  $p$ ;

$tr(u, w)$ : travel time from the location of job  $u$  to the location of job  $w$ ;

#### Decision variables:

We assume that insertion points are ordered in time within each gang. Notation  $i + 1$  represents the insertion point immediately after insertion point  $i$  ( $i$  and  $i + 1$  belong to the same gang). Given insertion point  $i$ , we call the remaining path at  $i$ , denoted as  $r_i$ , the segment of the original path between insertion points  $i$  and  $i + 1$ , see Fig. 11. In the figure, dotted and solid lines compose the new path, in which the former may have increased or shrunk, and the latter remain unchanged. Note that the path is shifted in time. In addition, we assume that before insertion point  $i$  no changes have been made to the path. Let also  $t(p, j), p \in \mathcal{P}, j \in \mathcal{N}$  be  $sd(p)$ -time of node  $j$ .

We then define 3 types of binary variables:

$$X_s^i = \begin{cases} 1 & \text{if path } s \in \mathcal{P} \text{ is inserted at the insertion point } i \in \mathcal{I} \\ 0 & \text{otherwise.} \end{cases}$$

$$Y_i^j = \begin{cases} 1 & \text{1 if } r_i, i \in \mathcal{I} \text{ starts at node } j \\ 0 & \text{otherwise.} \end{cases}$$

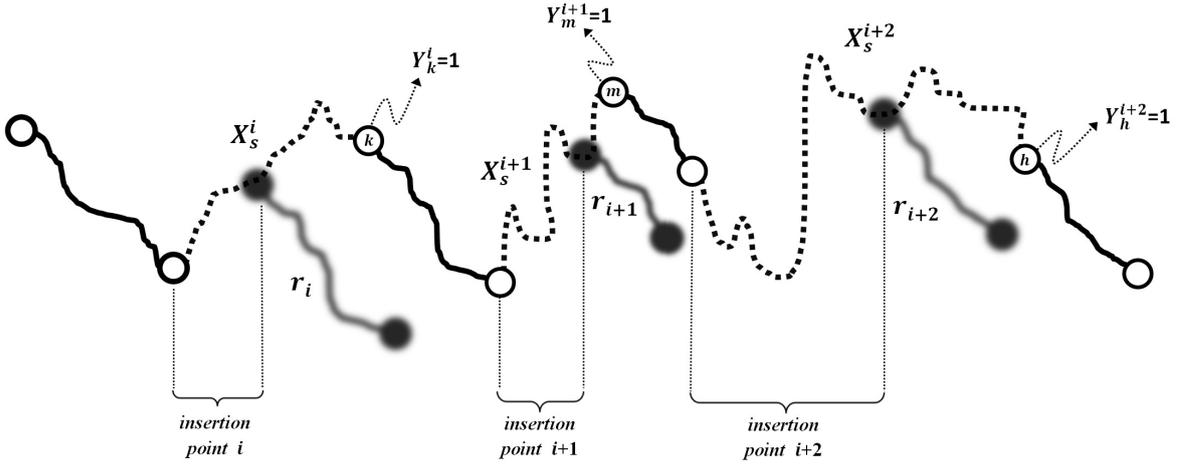


Figure 11 New path after insertion of unassigned/uncovered jobs.

$$Z_{uw} = \begin{cases} 1 & \text{1 if jobs } (u, w) \in Sim \text{ are performed at the same time} \\ 0 & \text{otherwise.} \end{cases}$$

## A.2. IP model

The mathematical model is formulated as follows:

Objective function:

$$\min \sum_{s \in \mathcal{P}} \sum_{i \in \mathcal{I}} \gamma_s^i X_s^i$$

Subject to:

**Node-sequence assignment:**

$$\sum_{\substack{s: u \in jobs(s) \\ s \in \mathcal{P}}} \sum_{i \in \mathcal{I}} X_s^i = 1, \forall u \in \mathcal{F}. \quad (1)$$

Constraint (1) imposes that each extracted/uncovered node belongs to exactly one reconstructed path  $s \in \mathcal{P}$ .

**Insertion constraint:**

$$\sum_{s \in \mathcal{P}} X_s^i \leq 1, \forall i \in \mathcal{I}. \quad (2)$$

Constraint (2) specifies that no more than one reconstructed path can be inserted at an insertion point.

**Starting point constraint:**

$$\sum_{j \in \mathcal{J}^i} Y_j^i = 1, \forall i \in \mathcal{I}. \quad (3)$$

Constraint (3) specifies that exactly one starting node is selected for insertion point  $i$ .

**Time constraints:**

$$X_s^i \leq \sum_{\substack{j \in \mathcal{J}^i \\ j = (fj(s), t) \\ t \geq ed(s) + tr(lj(s), fj(r_i))}} Y_j^i, \forall s \in \mathcal{P}, \forall i \in \mathcal{I}, \quad (4)$$

$$Y_j^i \leq \sum_{\substack{s \in \mathcal{P} \\ sd(s) \geq ed(r_i + t(r_i, j)) + tr(lj(r_i), fj(s))}} X_s^{i+1}, \forall j \in \mathcal{J}^i, \forall i \in \mathcal{I}. \quad (5)$$

Constraint (4) ensures that the starting time of  $r_i, i \in \mathcal{I}$  must be after the end time of the inserted path  $s$  at insertion point  $i$ . Constraint (5) imposes that the inserted path  $s$  at the next insertion point  $i + 1$  must start after the end time of  $r_i$  and the corresponding travel time.

**Precedence constraints:**

$$\sum_{\substack{s:(u,t) \in s \\ s \in \mathcal{P}}} \sum_{t \leq t_1} \sum_{i \in \mathcal{I}} X_s^i \leq \sum_{\substack{s:(w,t) \in s \\ s \in \mathcal{P}}} \sum_{t \geq t_1 + D_u} \sum_{i \in \mathcal{I}} X_s^i, \forall (u, w) \in Pre, u, w \in \mathcal{F}, \forall t_1 \in (0, T], \quad (6)$$

$$\sum_{\substack{s:(u,t) \in s \\ s \in \mathcal{P}}} \sum_{t \leq t_1} \sum_{i \in \mathcal{I}} X_s^i \leq \sum_{j \in \mathcal{J}^i} \sum_{\substack{i \in \mathcal{I} \\ (w,t) \in r_i + t(r_i,j)}} \sum_{t \geq t_1 + D_u} Y_j^i, \forall (u, w) \in Pre, u \in \mathcal{F}, w \in \mathcal{O}, \forall t_1 \in (0, T], \quad (7)$$

$$\sum_{\substack{i \in \mathcal{I} \\ (u,t) \in r_i + t(r_i,j)}} \sum_{t \leq t_1} \sum_{j \in \mathcal{J}^i} Y_j^i \leq \sum_{\substack{s:(w,t) \in s \\ s \in \mathcal{P}}} \sum_{t \geq t_1 + D_u} \sum_{i \in \mathcal{I}} X_s^i, \forall (u, w) \in Pre, u \in \mathcal{O}, w \in \mathcal{F}, \forall t_1 \in (0, T], \quad (8)$$

$$\sum_{\substack{i \in \mathcal{I} \\ (u,t) \in r_i + t(r_i,j)}} \sum_{t \leq t_1} \sum_{j \in \mathcal{J}^i} Y_j^i \leq \sum_{\substack{i \in \mathcal{I} \\ (w,t) \in r_i + t(r_i,j)}} \sum_{t \geq t_1 + D_u} \sum_{j \in \mathcal{J}^i} Y_j^i, \forall (u, w) \in Pre, u, w \in \mathcal{O}, \forall t_1 \in (0, T]. \quad (9)$$

Constraints (6)–(9) ensure that job  $u$  must be processed before job  $w$  for each  $(u, w) \in Pre$ . For example, in (6) we impose for each possible time  $t_1$ , if job  $u$  starts before  $t_1$ , then job  $w$  must start after  $t_1 + D_u$ .

**Robustness constraints:**

$$\sum_{\substack{s:(u,t) \in s \\ s \in \mathcal{P}}} \sum_{i \in \mathcal{I}} X_s^i + \sum_{\substack{s:(w,t) \in s \\ s \in \mathcal{P}}} \sum_{i \in \mathcal{I}} X_s^i \leq Z_{uw} + 1, \forall (u, w) \in Sim, u, w \in \mathcal{F}, \forall t \in [0, T], \quad (10)$$

$$\sum_{\substack{s:(u,t) \in s \\ s \in \mathcal{P}}} \sum_{i \in \mathcal{I}} X_s^i + \sum_{\substack{i \in \mathcal{I} \\ (w,t) \in r_i + t(r_i,j)}} \sum_{j \in \mathcal{J}^i} Y_j^i \leq Z_{uw} + 1, \forall (u, w) \in Sim, u \in \mathcal{F}, w \in \mathcal{O}, \forall t \in [0, T], \quad (11)$$

$$\sum_{\substack{i \in \mathcal{I} \\ (u,t) \in r_i + t(r_i,j)}} \sum_{j \in \mathcal{J}^i} Y_j^i + \sum_{\substack{s:(w,t) \in s \\ s \in \mathcal{P}}} \sum_{i \in \mathcal{I}} X_s^i \leq Z_{uw} + 1, \forall (u, w) \in Sim, u \in \mathcal{O}, w \in \mathcal{F}, \forall t \in [0, T], \quad (12)$$

$$\sum_{\substack{i \in \mathcal{I} \\ (u,t) \in r_i + t(r_i,j)}} \sum_{j \in \mathcal{J}^i} Y_j^i + \sum_{\substack{i \in \mathcal{I} \\ (w,t) \in r_i + t(r_i,j)}} \sum_{j \in \mathcal{J}^i} Y_j^i \leq Z_{uw} + 1, \forall (u, w) \in Sim, u, w \in \mathcal{O}, \forall t \in [0, T], \quad (13)$$

$$\sum_{u, w \in Sim} Z_{uw} \leq K. \quad (14)$$

Constraints (10)–(13) ensure that if jobs  $u$  and  $w$  are processed at the same time, then  $Z_{uw} = 1$ . Since robustness constraints are “soft”, we impose an upper bound (14).

This formulation/notation neglects job time windows and possible other requirements that are straight forward to incorporate. Soft precedence constraints can easily be derived from (6)–(9) and the concept presented in (10)–(13).